# Chapter 5
# Forces on Structures, Single-Wedge Sliding Analysis

## 5-1. General

Structures covered in this EM typically accommodate a difference in soil and/or water elevation over a short horizontal distance. The lateral forces applied to one side of the structure exceed those applied to the other side. The difference between these forces, and moments due to them, are resisted by forces developed at the base of the structure. These applied forces may be related to gravity, soil, water (including seepage when present), waves, wind, and earthquakes. The methods presented in this chapter are intended to produce reasonably conservative estimates of actual forces acting on a structure. The definitions of terms that will be used throughout this chapter are as follows:

*a. Single wedge.* The single wedge is the wedge to which forces are applied, i.e., the structure itself which is referred to as the structural wedge.

*b. Driving forces.* Driving forces are defined as those lateral (predominately horizontal) forces whose primary influence is to decrease structural stability. The side of the structure upon which these forces are applied will be called the driving side.

*c. Resisting forces.* Resisting forces are defined as those lateral (predominately horizontal) forces whose primary influence is to increase structural stability. The side of the structure upon which these forces are applied will be called the resisting side. The resisting side is on the opposite side of the structural wedge from the driving side. The difference between the driving and resisting forces is transferred to the foundation by the structural wedge.

## 5-2. Single-Wedge Stability Analyses

*a. Basic requirements.* For the single-wedge analysis, the engineer must calculate the driving and resisting earth forces, lateral water forces, and uplift and apply them to the structural wedge. The vertical drag force discussed in Appendix F may also be included, if the requirements stated in the appendix are satisfied. Using these forces and the weight of the structural wedge, the engineer can determine the magnitude, location, and slope of the resultant acting at the base of the structural wedge. The resultant will be used to check for sliding stability, and the location of the resultant will be used to determine the potential for partial loss of contact between the structure and the foundation materials. The resultant and its location will also be used to check against the bearing capacity of the foundation materials (see EM 1110-2-1903 and Chapter 5 of EM 1110-2-2502). The calculated forces applied to the structural wedge will also be used for design of the structural elements (e.g., bending in the stem of a retaining wall).

*b. Earth forces.* Lateral earth forces acting on the single wedge should be calculated using the minimum required factor of safety against sliding to obtain the developed soil strength parameters $\phi_d$ and $c_d$. These developed parameters shall be used in Equations 5-3 through 5-15 to calculate the lateral earth forces acting on the driving side, and used in Equations 5-16 through 5-22 to calculate the lateral earth forces acting on the resisting side. The values for the developed soil strength parameters shall be determined as:

$$\phi_d = \tan^{-1}\left(\frac{\tan\,\phi}{FS_{sliding}}\right) \qquad \text{and} \qquad c_d = \frac{c}{FS_{sliding}} \tag{5-1}$$

where

$FS_{sliding}$ = required factor of safety against sliding

*c. Sliding.* Separate the resultant into components parallel and normal to the base plane of the structure wedge. The sliding factor of safety is then calculated as follows:

$$FS = \frac{N \tan \phi + c L}{T} \tag{5-2}$$

where

$\tan \phi$, $c$ = full shear strength parameters for foundation

$N$ = the component of the resultant normal to the base

$T$ = the component of the resultant parallel to the base

$L$ = length of base in compression

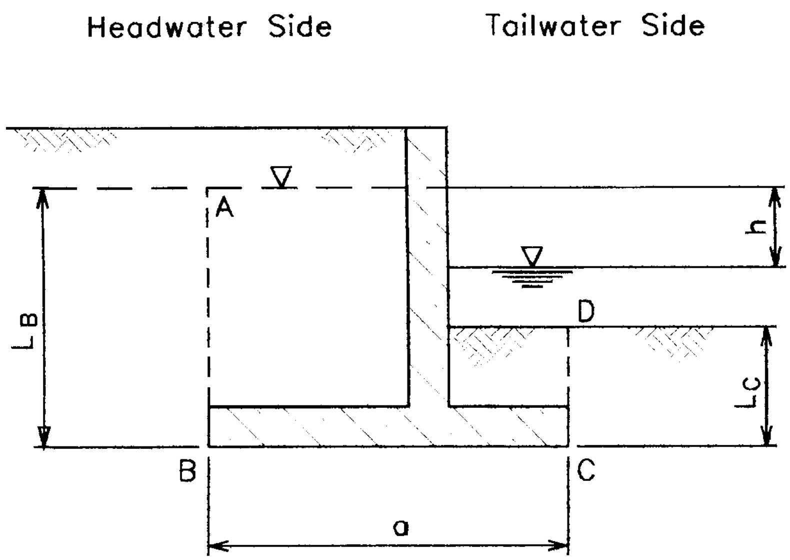
If the safety factor is equal to or greater than the required safety factor, sliding stability criteria is satisfied. Note that this calculated safety factor might not be equal to the minimum factor used to determine developed soil strength parameters. If there is a significant difference, it may be appropriate to re-evaluate the developed soil strength parameters used to determine soil forces. The resultant location shall be used to determine the length of the base that is in compression, and cohesion shall not be considered to be effective on that part of the base not in compression. If there is any loss of contact, uplift forces should be re-evaluated, except for seismic loading conditions.

## 5-3. Water Pressures

In all cases, water pressure at a point may be calculated by multiplying the pressure head at the point by the unit weight of water. As water has no shear strength, water pressures are equal in all directions. The pressure head at a point is the height water would rise in a piezometer placed at the point. The pressure head is equal to the total head minus the elevation head. The elevation head is the height of the point itself above an arbitrary datum. Total head-water pressures must be added to effective earth pressures to obtain total lateral pressures.

*a. Static pressures.* For static water (no seepage) above or below the ground surface, the total head is constant and the pressure head at any point is the difference in elevation between the water surface and the point of interest.

*b. Water pressures where seepage is present.* Where seepage occurs, the pressure heads at points of interest must be obtained from a seepage analysis. Where soil conditions adjacent to and below a structure can be assumed homogeneous (or can be mathematically transformed into equivalent homogeneous conditions), simplified methods such as the line-of-seepage method may be used. However, designers should ensure that water pressures are based on appropriate consideration of actual soil conditions. The line- of-seepage method is illustrated in Figure 5-1. The total heads at the ends of the base (points B and C) are estimated by assuming that the total head varies linearly along the shortest possible seepage path (ABCD). Once the total heads at B and C are known, the uplift pressures $u_B$ and $u_C$ are calculated by subtracting the elevation head from the total head at each point and multiplying the resultant pressure head by the unit weight of water. Where a key is present (Figure 5-2), point B is at the bottom of the key, and line BC is drawn diagonally. Permeabilities that are different in the horizontal and vertical directions can be handled by adjusting the length of the different segments along the total seepage path in accordance with the relationship between




L = Total length of seepage path = $L_B + a + L_C$

h = Head difference across wall

$u_B$ = Water pressure at B = $\left[L_B - \frac{h\,L_B}{L}\right]\gamma_w$

$u_C$ = Water pressure at C = $\left[L_B - \frac{h(L_B + a)}{L}\right]\gamma_w$

Where $\gamma_w$ = The unit weight of water

**Figure 5-1. Line of seepage method for water pressures**

these different permeabilities. A drainage system should be considered for all retaining walls. The benefits of a drainage system are the elimination of excess hydrostatic pressures in the backfill, lower lateral water pressures on the structure, and a reduction of uplift pressures. The efficiency of drains should be determined in accordance with provisions in Chapters 3 and 4.

*c. Sheet-pile cutoff.* Steel sheet-pile cutoffs are not entirely watertight due to leakage at the interlocks, but can significantly reduce the possibility of piping of coarse-grained foundation material. The efficiency of a steel sheet-pile cutoff through a coarse-grained stratum in reducing uplift depends upon conditions at the interlocks, the penetration distance (*P*) of the cutoff into the pervious stratum, and the depth (*D*) below the base of the structure to the top of impervious material. When $P \geq 0.95D$, and the pile interlocks are in good condition, an efficiency (*E*) of 0.50 may be assumed (EM 1110-2-2502 and Soils Project Item No. CW-460A). It has been observed, that steel sheet piling driven into certain types of foundation material (such as gravel) has a complete loss of interlock. Before relying on a steel sheet-pile cutoff the designer must be certain that the assumed reduction in seepage will actually occur. A design uplift diagram and method for computing pressures at pertinent points are shown in Figure 5-3. In this figure, uplift pressure for that part of the base on the heel side of the cutoff is that due to the full head (no seepage). Seepage is assumed to occur between the cutoff and the toe of the structure, and soil permeabilities are assumed equal in the horizontal and vertical directions.

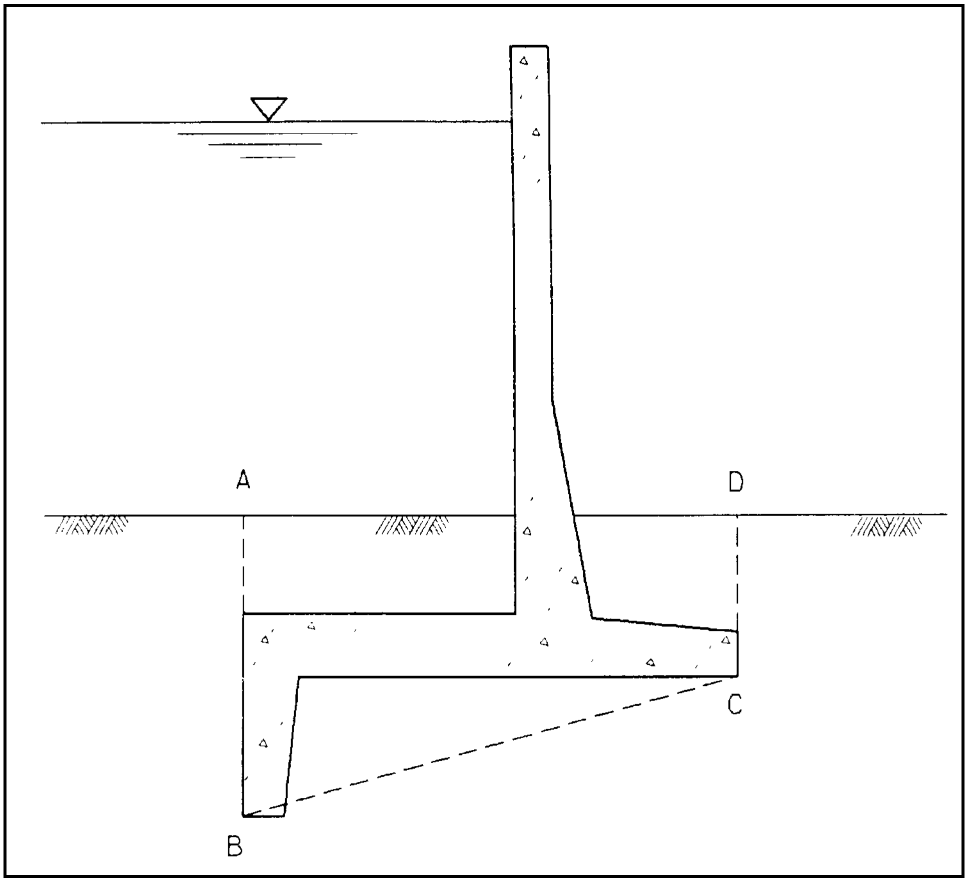



Figure 5-2. Seepage path for wall with key

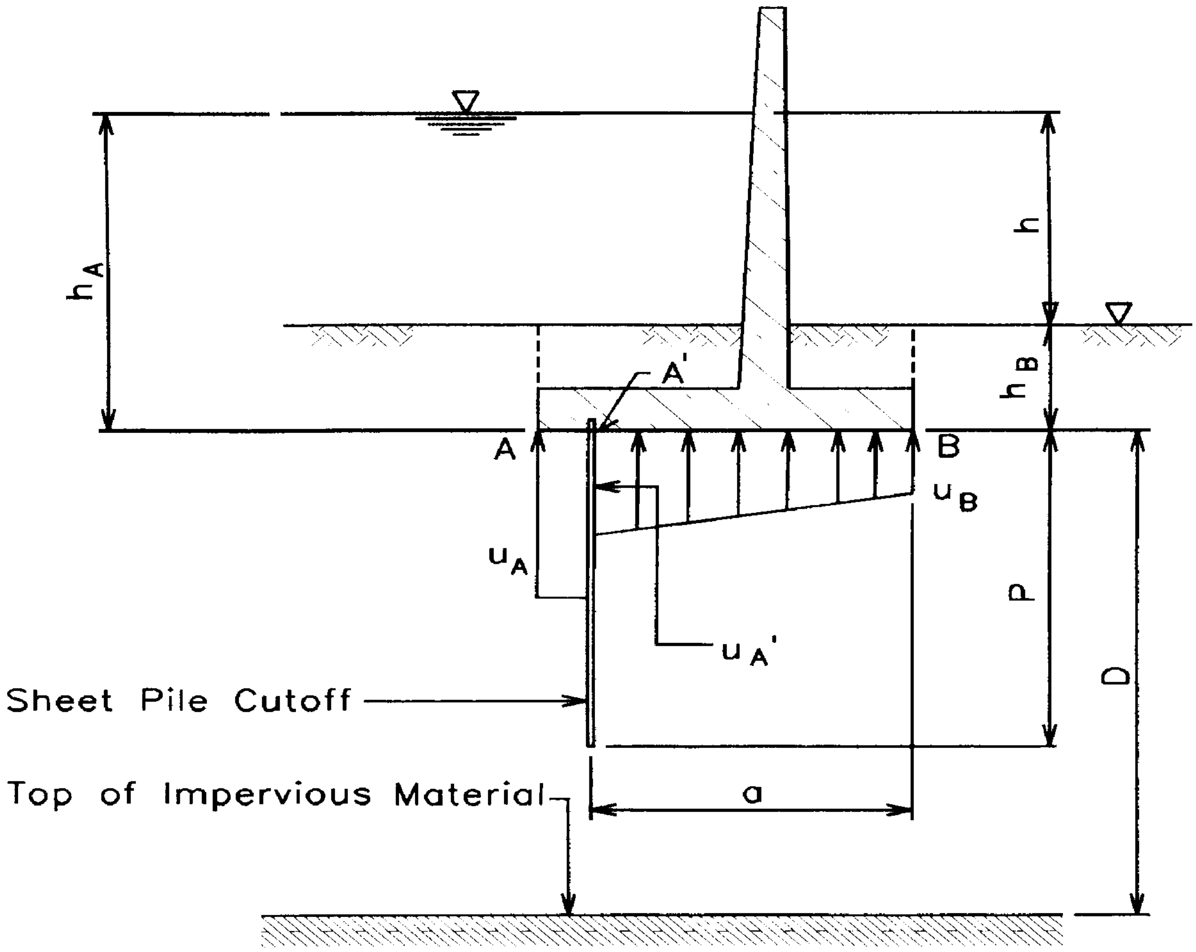
*d. Uplift calculation for rock foundations.* Seepage beneath structures founded on rock typically occurs in joints and fractures, not uniformly through pores as assumed for soils. Consequently, the assumptions of isotropy and homogeneity and the use of two-dimensional analysis models employed for soil foundations will generally be invalid. Total head, uplift pressure, and seepage quantities may be highly dependent on the type, size, orientation, and continuity of joints and fractures in the rock and the type and degree of treatment afforded the rock foundation during construction. Since any joints or fractures in the rock can be detrimental to seepage control, the joints should be cleaned out and filled with grout before structural concrete is placed. For structures on rock, the total seepage path can be assumed to be length of base that is in compression. For a more detailed discussion of uplift pressures for structures on rock foundations, see Chapter 4. See Appendix C for illustrations.

## 5-4. Earth Pressures and Forces

*a. Active earth pressures.*

(1) Cohesionless backfill. Cohesionless materials such as clean sand are the recommended backfill for most structures. Large-scale tests (Terzaghi 1934; Tschebatarioff 1949; Matsuo, Kenmochi, and Yagi 1978) with cohesionless (c = 0) backfills have shown that lateral pressures are highly dependent on the magnitude and direction of wall movement. The minimum lateral pressure condition, or active earth pressure, develops when a structure rotates about its base and away from the backfill an amount on the order of 0.001 to 0.005 radians. As the structure moves, horizontal stresses in the soil are reduced, and vertical stresses due to backfill weight are resisted by increasing shear stresses until shear failure is imminent (Figures 5-4 and 5-5).

(2) Cohesive backfill. For situations where cohesive backfill is unavoidable, solutions are included herein for earth pressures involving both tan ϕ and *c* soil strength parameters. Where cohesive backfill is used, two analyses (short-term and long-term) are usually required in order to model conditions that may arise during the life of the structure. Short-term analyses modelconditions prevailing before pore water dissipation occurs, such as the end-of-construction condition. Unconsolidated-undrained (*Q*) test parameters, which yield a relatively high *c* value and a low or zero ϕ




$u_A$ = uplift at A = $\gamma_w h_A$

$u_B$ = uplift at B = $\gamma_w h_B + \dfrac{h_B}{a+h_B}(u_{A'} - \gamma_w h_B)$

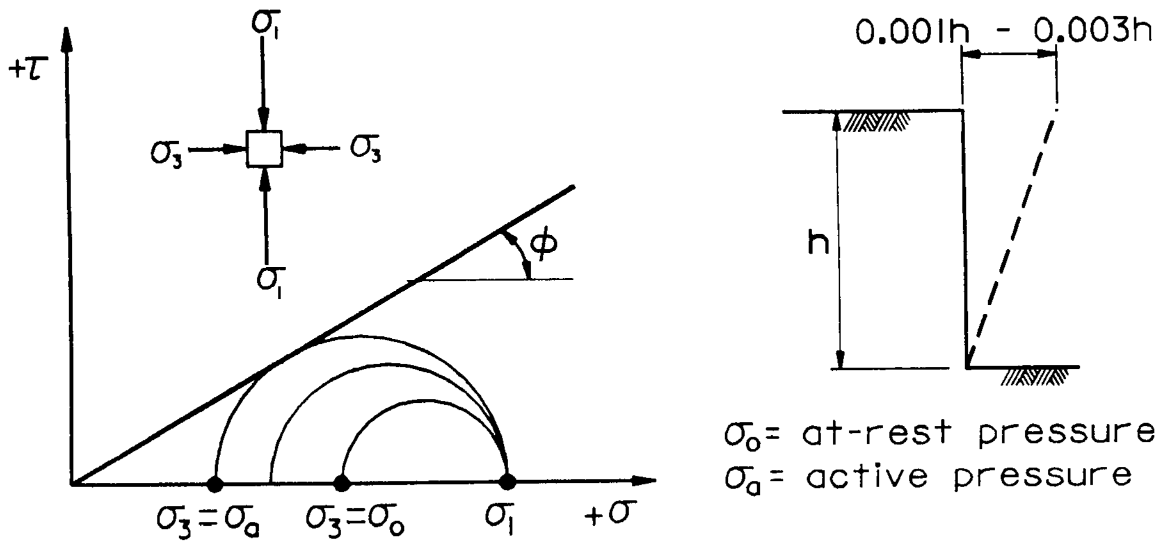
$u_{A'}$ = uplift at A' = $u_A - E(u_A - \gamma_w h_B)$

Where:

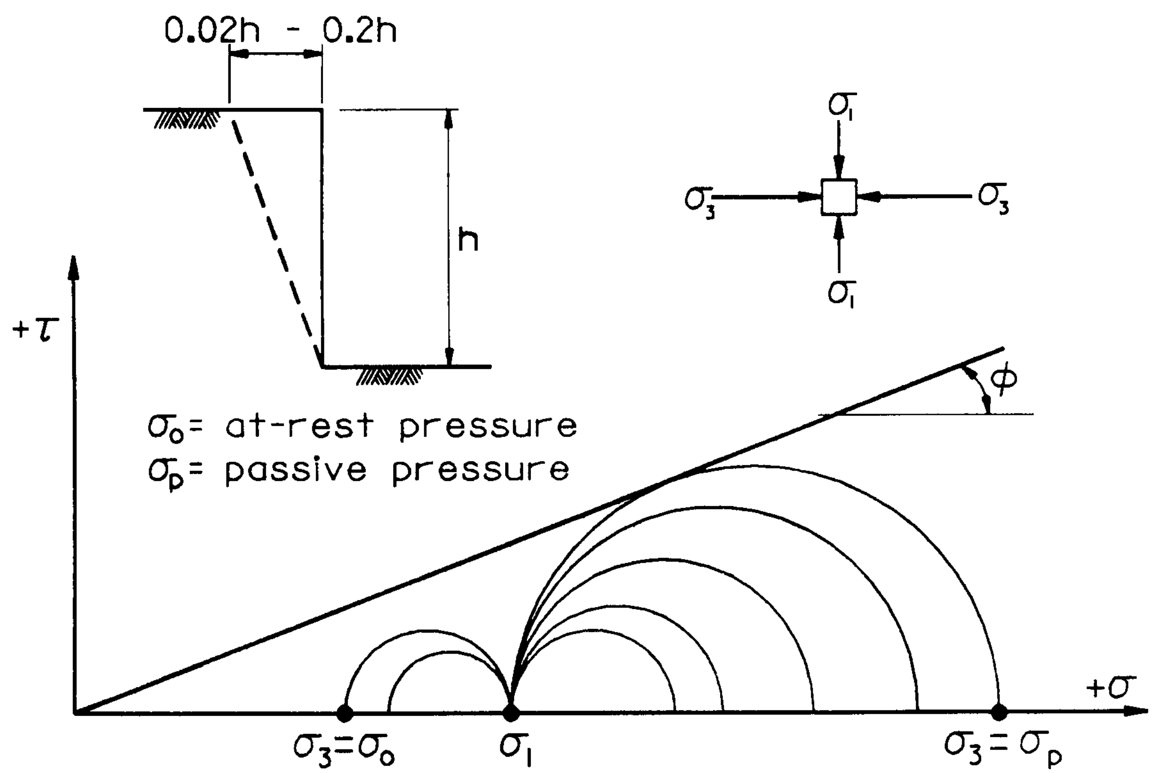
$\gamma_w$ = The unit weight of water

E = Efficiency of sheet pile cutoff

Figure 5-3. Uplift pressures with sheet pile cutoff




a. Development of active earth pressure




b. Development of passive earth pressure

**Figure 5-4. Development of earth pressures for a cohesionless material**

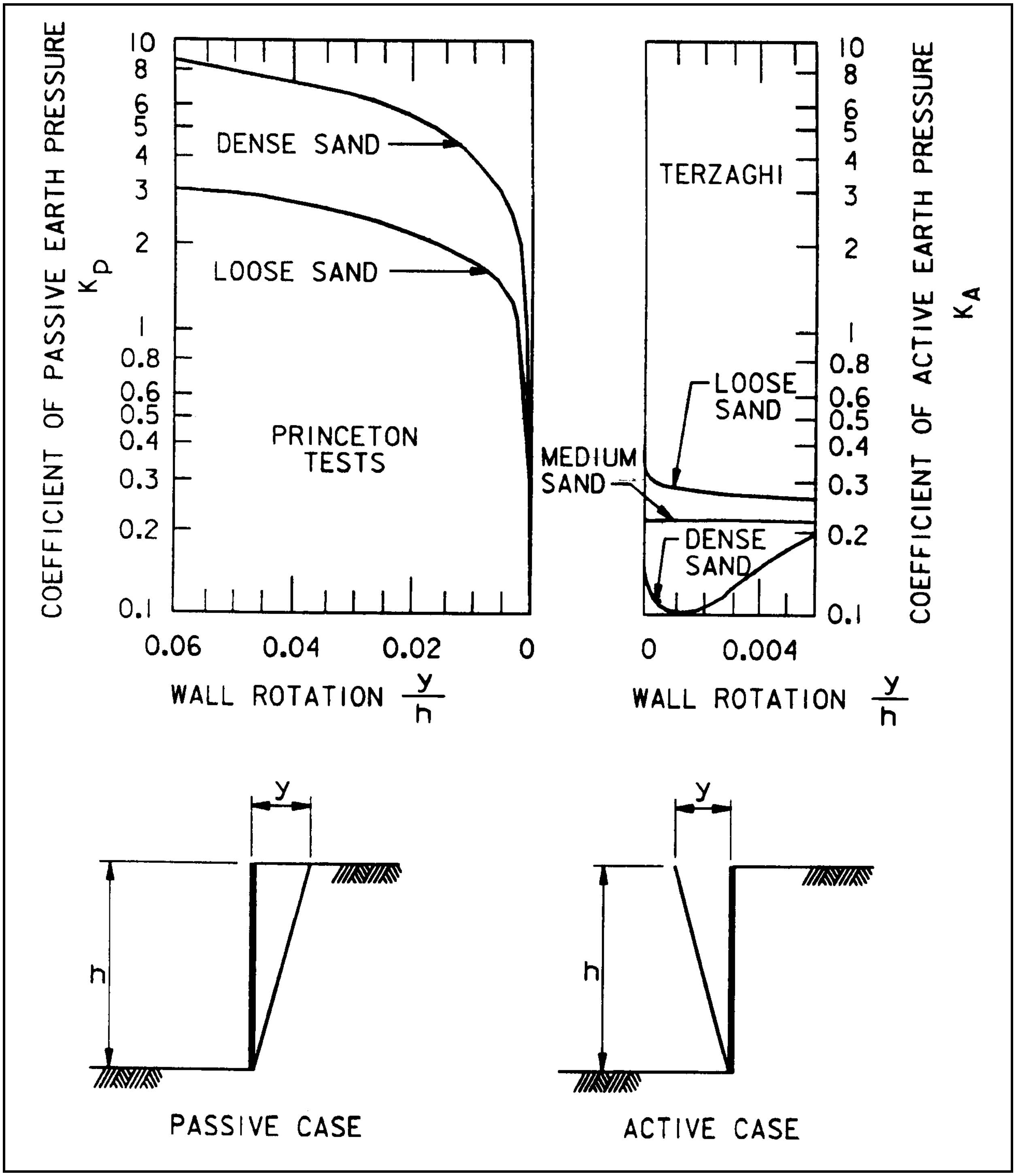



**Figure 5-5. Relationship of earth pressures to wall movements (after Dept. of Navy 1982b)**

value, are appropriate for short-term analyses. Short-term stability analyses should include the effects applied stress along shear planes is resisted by both shear stress and vertical stress components, higher lateral of water pressure in tension (cohesive) cracks in the backfill. Long-term analyses model conditions prevailing after shear-induced pore pressures have dissipated. For long-term analyses, consolidated-drained (*S*) test parameters are appropriate. These tests usually yield a relatively high value for $\phi$ and a low or zero value for $c$.

*b. Passive earth pressures.* If a structure is moved toward the backfill, lateral earth pressures increase and shear stresses reverse direction, first decreasing and then increasing to a maximum at failure (Figure 5-4). Because the pressures than those for the active case can be developed. Full development of passive pressure requires much larger structure rotations than those required for the active case, as much as 0.02 to 0.2 radians (Figures 5-4 and 5-5). However, the rotation required to develop one-half of the passive pressure is significantly less, as little as 0.005 radians. The designer must be certain that soil on the resisting side of any structure will always remain in place and not be excavated or eroded before its effect is included in the stability analyses.

*c. At-rest earth pressure.* If no structural movement occurs, the at-rest condition exists.

*d. Design earth pressures - driving side.* In practice, the active and passive earth pressure conditions seldom exist. Hydraulic structures in particular are designed using conservative criteria that results in relatively stiff structures. Structures founded on rock or stiff soils usually do not yield sufficiently to develop active pressures. Even for foundations capable of yielding, experiments with granular backfill (Matsuo, Kenmochi, and Yagi 1978) indicate that following initial yield and development of active pressures, lateral pressures may in time return to at-rest values. Another reference (Casagrande 1973) states that the gradual buildup of the backfill in compacted lifts produces greater-than-active pressures as do long-term effects from vibrations, water level fluctuations, and temperature changes. For these reasons and because such large rotations are required for the development of passive pressures, earth pressures on both the driving side and the resisting side of the single wedge (the structure) will be estimated by using the developed soil strength parameters ($\tan \phi_d$ and $c_d$), as defined in paragraph 5-2. These parameters are then used in the equations required to calculate the equivalent fluid earth-pressure coefficients ($K$).

(1) General wedge method for equivalent fluid pressure coefficients. Lateral forces between earth wedges are assumed to act horizontally, except that they are assumed to act parallel to the top surface of driving side wedges when the surface slopes toward the structure. Equivalent fluid-pressure coefficients are calculated as follows (derivations for Equations 5-3 through 5-6 are provided in Appendix E):

$$\alpha = \tan^{-1}\left(\frac{C_1 + \sqrt{C_1^2 + 4C_2}}{2}\right) \tag{5-3}$$

where

$\alpha$ = the critical slip plane angle for the earth wedge

$$C_1 = \frac{2(\tan\phi_d + \tan\delta)\tan\phi_d - \left(\dfrac{4V}{\gamma(h^2 - d_c^2)}\right)(1 + \tan^2\phi_d)\tan\beta + \left(\dfrac{4c_d}{\gamma(h + d_c)}\right)s}{A} \tag{5-4}$$

$$C_2 = \frac{t + \left(\frac{2V}{\gamma(h^2 - d_c^2)}\right)(1 + \tan^2\phi_d)\tan^2\beta + \left(\frac{2c_d}{\gamma(h + d_c)}\right) r}{A} \tag{5-5}$$

$$A = \tan\phi_d + \tan\delta - \left(\frac{2V}{\gamma(h^2 - d_c^2)}\right)(1 + \tan^2\phi_d) + \left(\frac{2c_d}{\gamma\,(h + d_c)}\right) r \tag{5-6}$$

where

$r$ = 1 - tanδ tan$\phi_d$ - tanβ (tanδ + tan$\phi_d$)

$s$ = tanβ + tan$\phi_d$ + tanδ (1 - tanβ tan$\phi_d$)

$t$ = tan$\phi_d$ - tanβ - (tanδ + tanβ) tan$^2\phi_d$

and

ϕ = soil internal friction parameter

$\phi_d$ = developed internal friction parameter

$c$ = soil cohesion parameter

$c_d$ = developed cohesion parameter

ß = top surface slope angle, positive when slope is upward when moving away from the structure

δ = wall friction angle (use δ = β). Vertical shear (drag), as discussed in Appendix F, shall not be used to calculate the value of α or equivalent fluid earth-pressure coefficients. However, drag may be used in addition to lateral earth pressures when the requirements of Appendix F are satisfied

γ = average unit weight of soil (moist weight above water table, buoyant weight below)

$V$ = strip surcharge

$h$ = height of vertical face of earth wedge

$d_c$ = depth of cohesion crack in soil (should always be assumed filled with water when calculating lateral forces)

When the top surface of the backfill is broken, solutions for α may be obtained by using analogous positive and negative strip surcharges. Examples are presented in Appendix D.

The equivalent fluid-pressure coefficient for cohesionless soils is then:

$$K = \frac{1 - \tan\phi_d \cot\alpha}{\cos\delta\,[(1 - \tan\delta\tan\phi_d) + (\tan\phi_d + \tan\delta)\tan\alpha]} \tag{5-7}$$

and for soils that possess cohesive properties as well as internal friction:

$$K_c = \frac{1}{2\cos^2\alpha\,(\tan\alpha - \tan\beta)\,[1 - \tan\delta\,\tan\phi_d + (\tan\delta + \tan\phi_d)\tan\alpha]} \tag{5-8}$$

The equation for the depth of a cohesive crack is:

$$d_c = \frac{2\,K_c\,c}{K\,\gamma\left(\dfrac{\tan\,\alpha}{\tan\,\alpha - \tan\,\beta}\right)} \tag{5-9}$$

And the total lateral earth force is calculated as:

$$P = \frac{1}{2}\,K\,\gamma\left(\frac{\tan\,a}{\tan\,\alpha - \tan\,\beta}\right)(h - d_c)^2 + K\,V\,\tan\,\alpha \tag{5-10}$$

When any of the variables in Equations 5-3 through 5-10 are not present in a particular problem, they are set equal to zero, thereby simplifying these equations. See Figure 5-6 for an illustration of a wedge containing cohesionless soil showing the methods used to calculate the lateral force, and the earth pressure at any point on the vertical face of the wedge. Figure 5-7 shows the methods used for calculating earth pressures and force for a wedge consisting of a soil possessing cohesive strength as well as internal friction. Figure 5-8 shows the method used to determine the pressure distribution for a strip surcharge (a line load *V*).

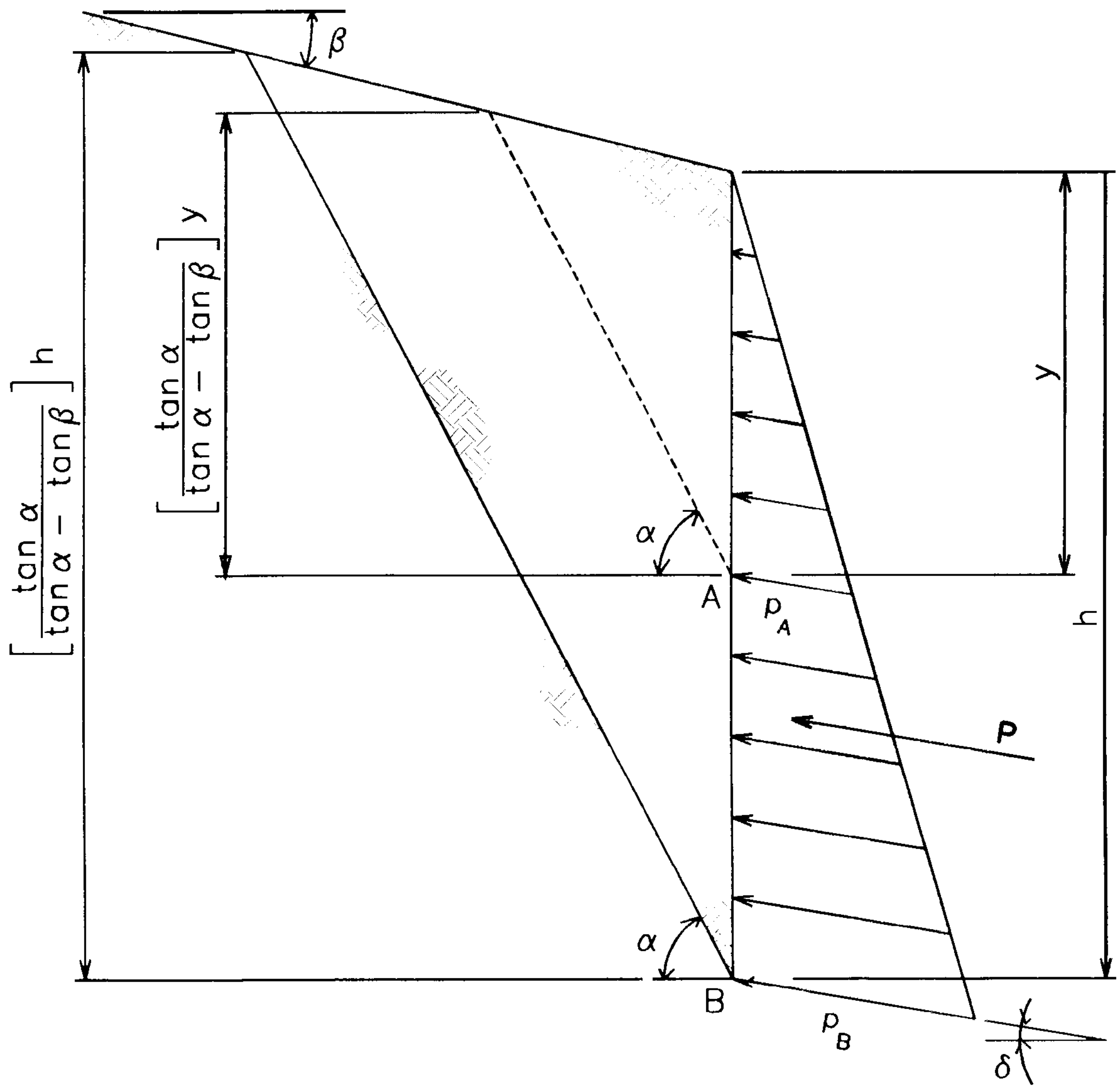
When ß is greater than $\phi_d$ a solution for α cannot be obtained from Equation 5-3 because the number under the radical will be negative, making the square root indeterminate. However, when ß is equal to $\phi_d$, Equation 5-3 will give a value for α equal to ß and $\phi_d$. It can be shown that (when $\alpha = \phi_d =$ ß) the total lateral earth force, for a granular soil not supporting a strip surcharge, becomes:

$$P = \frac{1}{2}\,\gamma\,h^2\,\cos\,\beta$$

This equation gives the maximum driving side lateral earth force that can occur and should be used when ß is equal to or greater than $\phi_d$.

(2) Equivalent fluid pressure coefficients from Coulomb's equation. Coulomb's equation may be used to calculate the equivalent fluid-pressure coefficient for near at-rest conditions, with certain limitations. These limitations are listed:

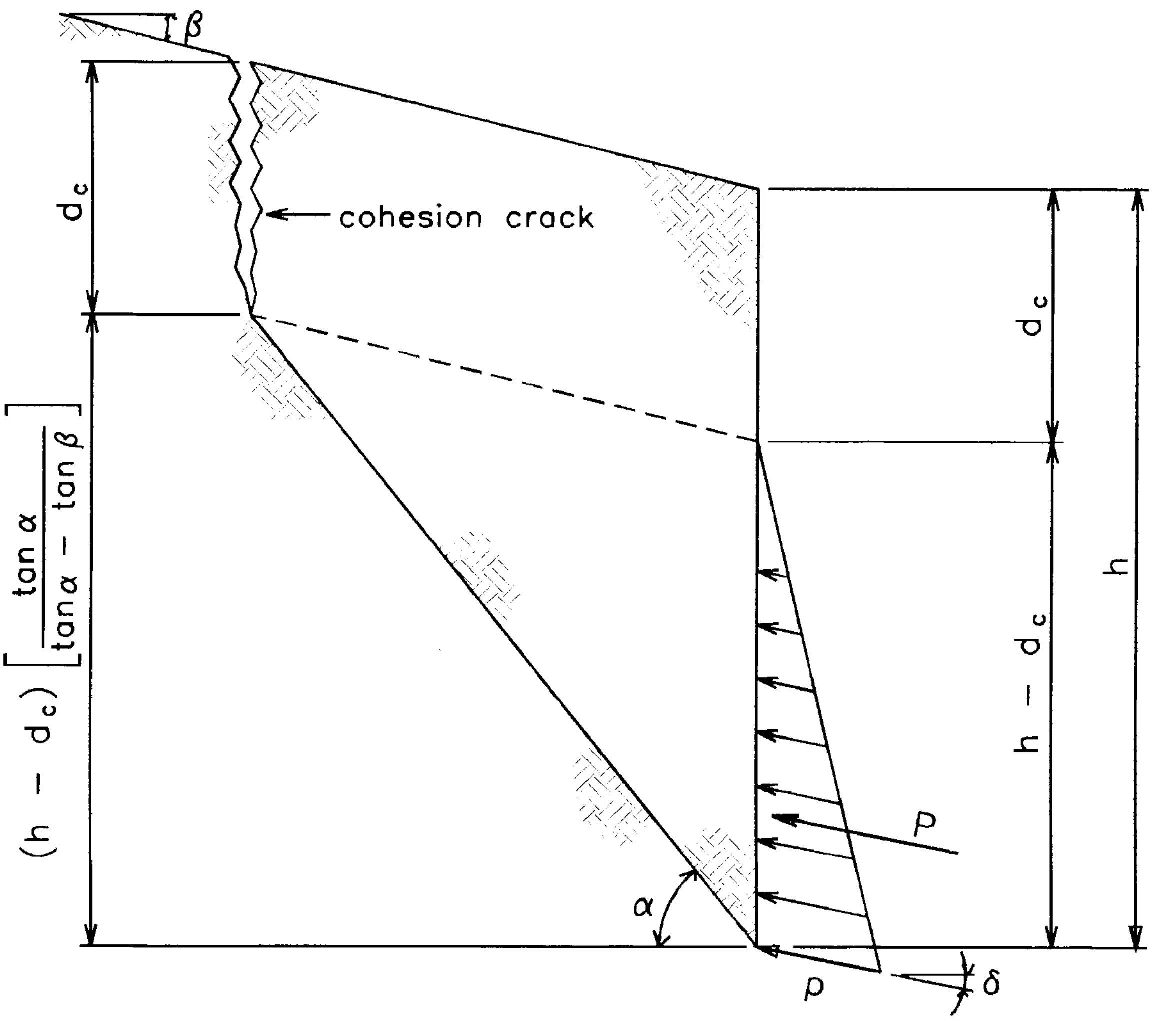
- There is only one soil material. (Material properties are constant.) There can be more than one soil layer if the top surfaces of all layers are horizontal.




$$p_A = K\gamma\left[\frac{\tan\alpha}{\tan\alpha - \tan\beta}\right]y \qquad p_B = K\gamma\left[\frac{\tan\alpha}{\tan\alpha - \tan\beta}\right]h$$

$$P = 1/2\,K\gamma\left[\frac{\tan\alpha}{\tan\alpha - \tan\beta}\right]h^2$$

**Figure 5-6. Calculation of earth pressures for cohesionless soil**

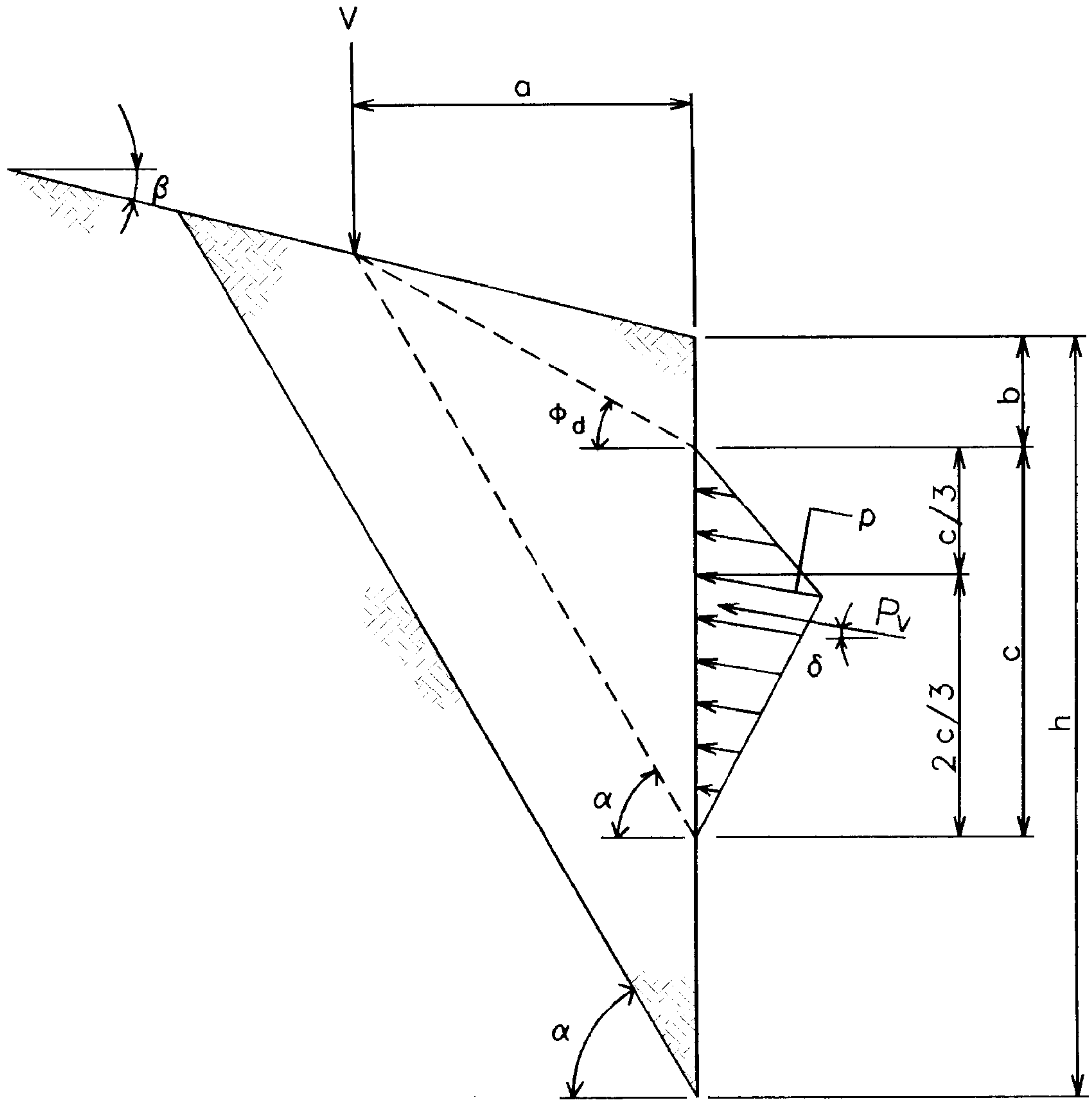



$$p = K\gamma\left[\frac{\tan\alpha}{\tan\alpha - \tan\beta}\right](h - d_c)$$

$$P = 1/2\ K\gamma\left[\frac{\tan\alpha}{\tan\alpha - \tan\beta}\right](h - d_c)^2$$

**Figure 5-7. Calculation of earth pressures for cohesive soil**

- The backfill surface is planar. (It may be inclined.)
- The water table is either completely above or completely below the earth wedge, unless the top surface of the wedge is horizontal. When the top surface is horizontal, the water table may lie within the wedge.
- Any surcharge is uniform and covers the entire top surface of the wedge.




$$b = a\,(\tan\Phi_d - \tan\beta)$$

$$c = a\,(\tan\alpha - \tan\Phi_d)$$

$$P_v = K\,V\,\tan\alpha$$

$$p = \frac{2P_v}{c}$$

**Figure 5-8. Earth pressure distribution for strip surcharge (line load)**

- The soil is cohesionless, unless the top surface is horizontal, in which case the soil may possess cohesive strength.

The equation for the equivalent fluid-pressure coefficient, for near at-rest conditions, using the Coulomb Equation is:

$$K = \frac{\cos^2 \phi_d}{\cos \beta \left[ 1 + \frac{\sqrt{\sin(\phi_d + \beta) \sin(\phi_d - \beta)}}{\cos^2 \beta} \right]^2} \tag{5-11}$$

And the total lateral earth force is:

$$P = \frac{1}{2} K \gamma h^2 \tag{5-12}$$

(3) Equivalent fluid pressure coefficients for simple conditions. When the top surface of the wedge is horizontal, planar, supports a uniform surcharge covering the entire top surface, and the soil possesses cohesive strength as well as internal friction, the equivalent fluid pressure coefficients in the preceding equations reduce to the following simple expressions:

$$K = \frac{1 - \sin \phi_d}{1 + \sin \phi_d} = \tan^2 \left(45° - \frac{\phi_d}{2}\right) \tag{5-13}$$

and

$$K_c = \sqrt{K} \tag{5-14}$$

and the depth of the cohesion crack becomes

$$d_c = \frac{2 c_d}{\gamma \sqrt{K}} \tag{5-15}$$

*e. Design earth pressures - resisting side.* Developed passive resistance may be included in the single-wedge sliding analysis by using soil strength parameters ($\tan \phi_d$ and $c_d$) that are obtained by dividing the full value of these parameters by the required factor of safety for sliding. In this manual, passive earth pressures and forces are assumed to act horizontally (wall friction angle $\delta = 0$). The equivalent fluid-pressure coefficient for passive earth pressure is calculated as follows:

$$\alpha = \tan^{-1} \left[ \frac{-C_1 + \sqrt{C_1^2 + 4C_2}}{2} \right] \tag{5-16}$$

$$C_1 = \frac{2\tan^2\phi_d - \dfrac{4V}{\gamma h^2}\left[\tan\beta\,(1 + \tan^2\phi_d)\right] + \dfrac{4c_d}{\gamma h}(\tan\phi_d - \tan\beta)}{A} \tag{5-17}$$

$$C_2 = \frac{\tan\phi_d(1+\tan\phi_d\tan\beta) + \tan\beta + \dfrac{2c_d\,(1+\tan\phi_d\tan\beta)}{\gamma h} - \dfrac{2V\tan^2\beta\,(1+\tan^2\phi_d)}{\gamma h^2}}{A} \tag{5-18}$$

where

$$A = \tan\phi_d + \frac{2c_d\,(1 + \tan\phi_d\tan\beta)}{\gamma h} + \frac{2V\,(1 + \tan^2\phi_d)}{\gamma h^2} \tag{5-19}$$

Then the equivalent fluid-pressure coefficients for passive pressure are:

$$K_P = \frac{1 + \tan\phi_d\cot\alpha}{1 + \tan\beta\tan\phi_d - (\tan\phi_d - \tan\beta)\tan\alpha} \tag{5-20}$$

and when cohesion is present:

$$K_{cP} = \frac{1}{2\cos^2\alpha\,(\tan\alpha - \tan\beta)\left[1 + \tan\beta\tan\phi_d - (\tan\phi_d - \tan\beta)\tan\alpha\right]} \tag{5-21}$$

The developed passive resistance is then:

$$P_P = \frac{1}{2}K_P\,\gamma\left(\frac{\tan\alpha}{\tan\alpha - \tan\beta}\right)h^2 + 2K_{cP}\,c_d\,h + K_P\,V\tan\alpha \tag{5-22}$$

Since cohesive cracking will not occur on the resisting side, the term for the depth of cohesive cracking ($d_c$) is not included in Equations 5-17 through 5-22.

## 5-5. Earth Pressures When Water Table Lies Within or Above Top of Backfill Wedge

Pressures and forces due to soil and water must be calculated separately, since wall friction ($\delta$) is not applicable to water pressure and the equivalent fluid pressure coefficients ($K$ and $K_c$) are for calculating lateral soil pressure only. $K$ for water is always equal to one. However, the effective unit weight of soil below the water table is affected by the uplift due to water. In lateral earth-pressure calculations, the moist unit weight of soil is used above the water table, and the buoyant unit weight is used below the water table. When calculating lateral water pressure and uplift, the effect of seepage (if it occurs) must be considered. Lateral pressures and forces when water and soil occur in combination are as follows:

$$\gamma_b = \frac{\gamma_s h_s - u}{h_s} \tag{5-23}$$

where

$\gamma_m$ = moist unit weight of soil, use above water table

$\gamma_s$ = saturated unit weight of soil

$\gamma_b$ = buoyant unit of weight of soil, use below water table

$h$ = height of vertical face of earth wedge

$h_S$ = height of water table above bottom of wedge

$u$ = lateral water pressure and uplift at bottom of earth wedge (consider effects of seepage when it occurs)

$p_s$ = lateral earth pressure at water table

$p$ = lateral earth pressure at bottom of earth wedge

$$p_s = K \gamma_m \left( \frac{\tan \alpha}{\tan \alpha - \tan \beta} \right) (h - h_s) \tag{5-24}$$

$$p = K \left[ \gamma_m h \left( \frac{\tan \alpha}{\tan \alpha - \tan \beta} \right) - (\gamma_m - \gamma_b) h_s \right] \tag{5-25}$$

Then the total lateral earth force is:

$$P = \frac{1}{2} p_s (h - h_s) + \frac{1}{2} (p_s + p) h_s \tag{5-26}$$

and the horizontal water force is:

$$P_w = \frac{1}{2} u h_s \tag{5-27}$$

See Figure 5-9 for an illustration.

## 5-6. Earthquake Forces (Mononobe-Okabe Type Analysis)

The pseudo-static approach presented here should only be used as a screening tool to determine whether or not a more precise dynamic analysis (Chapter 4) is necessary. The Mononobe-Okabe equations are an extension of the Coulomb equations that take into account the horizontal and vertical inertial forces acting on the soil. The same limitations that apply to Coulomb's equations (see paragraph 5-5.*d.*(2)) also apply to the Mononobe-Okabe equations. In order to cover all the conditions that might be encountered in practice, equations developed from the general wedge theory are

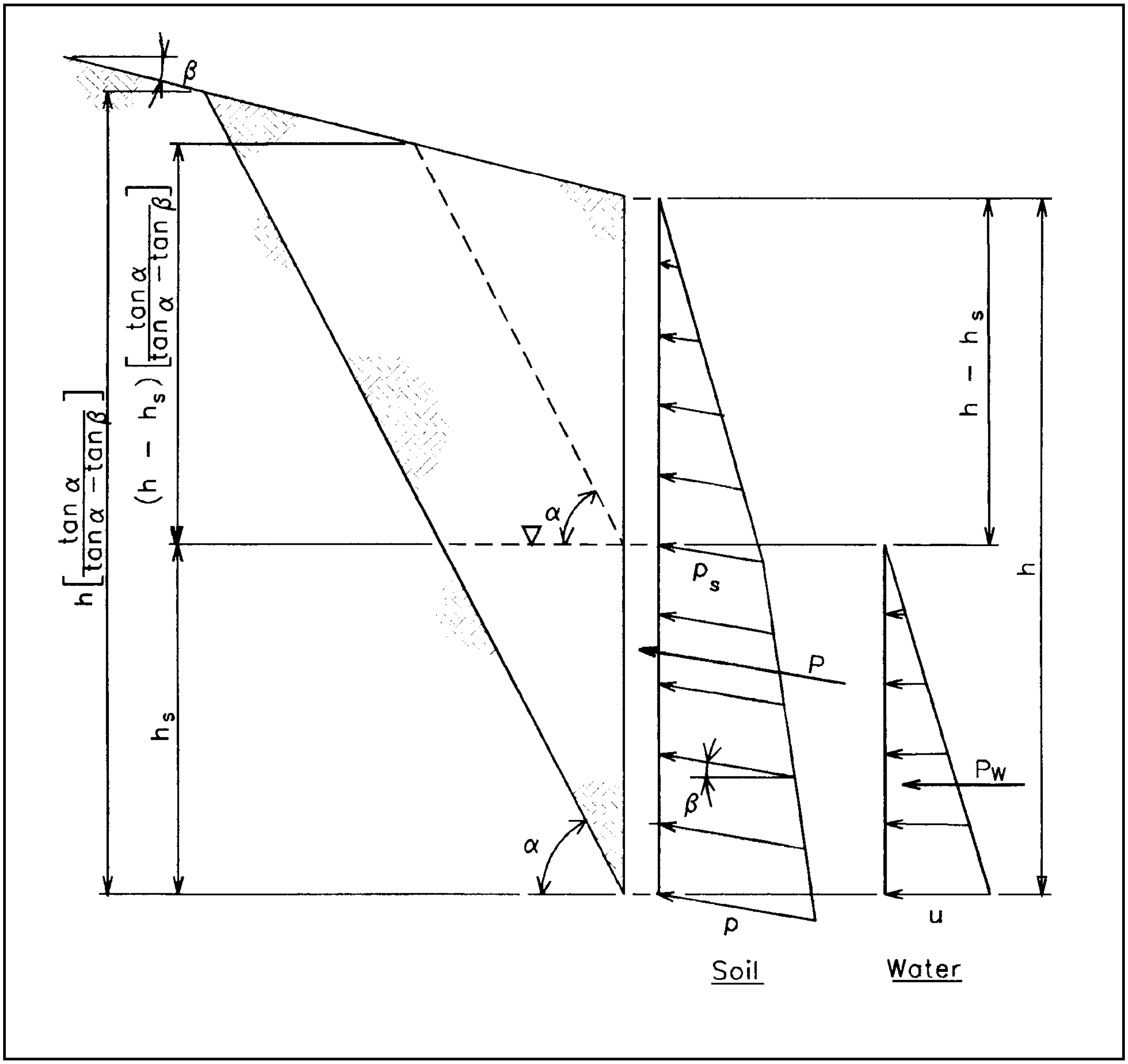



**Figure 5-9. Lateral soil pressure and water pressure combined**

presented in this EM. The equations given will produce the same results as the Mononobe-Okabe equations for those limited conditions where Mononobe-Okabe equations are applicable. The Mononobe-Okabe analysis is described in detail by Seed and Whitman (1970) and Whitman and Liao (1985). For the analysis, it is assumed that the full-active and full-passive failure conditions for the soil will develop during an earthquake, and the full value of the soil strength parameters (tan ϕ and *c*) will be used. Horizontal acceleration coefficients ($k_h$) for the United States and its territories can be obtained by using the spectral response maps provided in ER 1110-2-1806. Procedures for using the spectral response maps to obtain horizontal seismic coefficients are described in guidance memoranda. Vertical acceleration can be neglected. When $k_h$ exceeds 0.2, this analysis results in structures that are excessively large. Therefore, $k_h$ will be limited to 0.2. Driving and resisting forces for backfill possessing both cohesion and internal-friction strength parameters, with a sloping planar surface, a water table, and with δ and vertical acceleration ($k_V$) equal to zero can be calculated as follows:

*a. Driving force.* Driving wedge earthquake soil and water pressures are illustrated in Figure 5-10a. The static and dynamic components for a driving side wedge are as follows:

$p_s$ = lateral static earth pressure at the water table

$p$ = lateral static earth pressure at the bottom of the wedge

$$p_s = K \gamma_m \left( \frac{\tan \alpha}{\tan \alpha - \tan \beta} \right) (h - d_c - h_s) \tag{5-28}$$

$$p = K \left[ \gamma_m (h - d_c) \left( \frac{\tan\alpha}{\tan \alpha - \tan \beta} \right) - (\gamma_m - \gamma_b) h_s \right] \tag{5-29}$$

Then the total lateral static earth force is:

$$P = \frac{1}{2} p_s (h - d_c - h_s) + \frac{1}{2} (p_s + p) h_s \tag{5-30}$$

where

$$K = \frac{1 - \tan \phi \cot \alpha}{1 + \tan \phi \tan \alpha} \tag{5-31}$$

$$K_c = \frac{1}{2 \cos^2 \alpha (\tan \alpha - \tan \beta) (1 + \tan \phi \tan \alpha)} \tag{5-32}$$

$$d_c = \frac{2 K_c c}{K \gamma_m} \tag{5-33}$$

and

$$\alpha = \tan^{-1} \left( \frac{C_1 + \sqrt{C_1^2 + 4 C_2}}{2} \right) \tag{5-34}$$

$$C_1 = \frac{2 \tan \phi (\tan \phi - k_h) + \dfrac{4 c (\tan \phi + \tan \beta)}{\gamma (h + d_c)}}{A} \tag{5-35}$$

$$C_2 = \frac{\tan \phi (1 - \tan \phi \tan \beta) - (\tan \beta + k_h) + \dfrac{2 c (1 - \tan \phi \tan \beta)}{\gamma (h + d_c)}}{A} \tag{5-36}$$

$$A = (1 + k_h \tan \phi) \tan \phi + \frac{2 c (1 - \tan \phi \tan \beta)}{\gamma (h + d_c)} \tag{5-37}$$

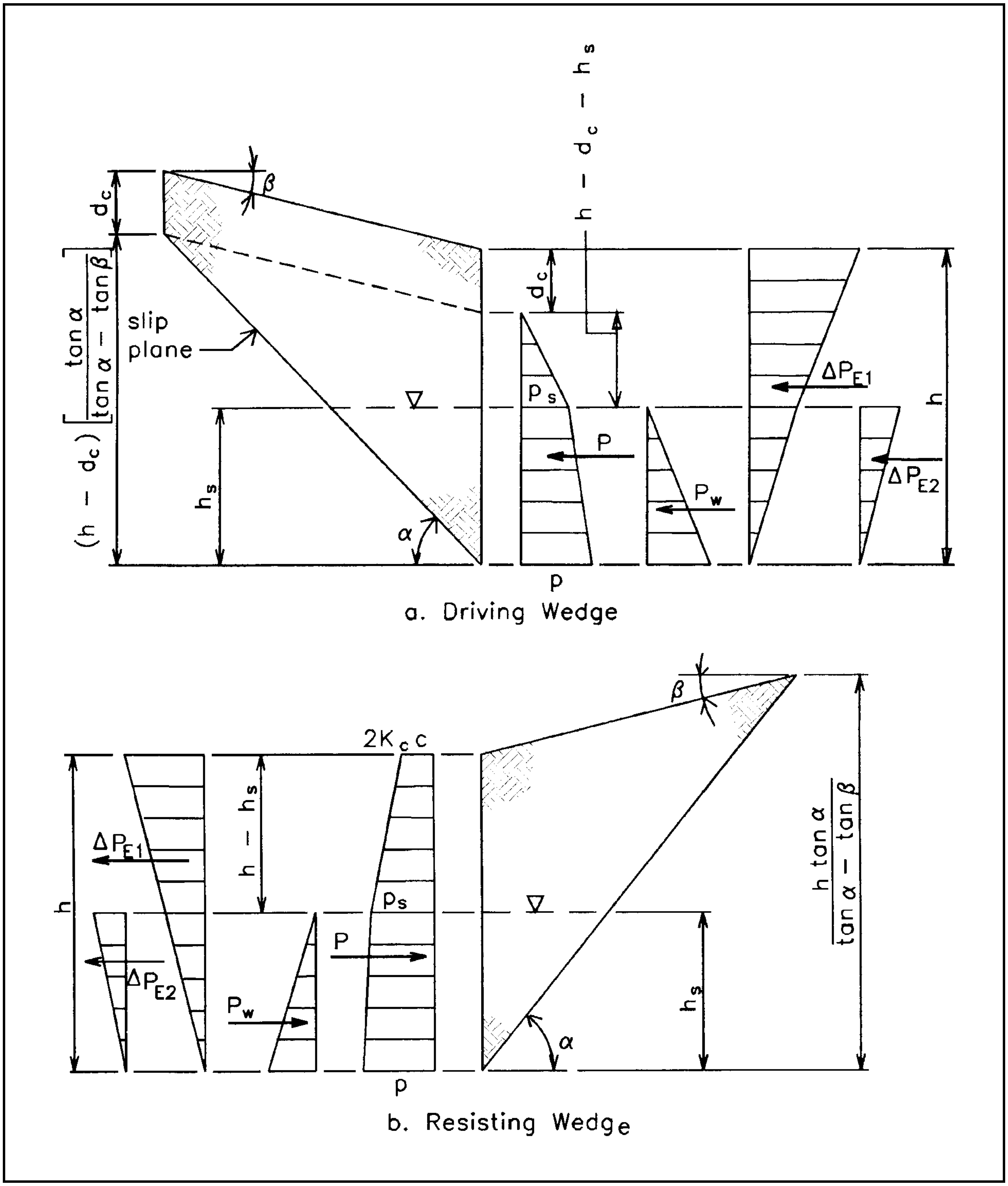



**Figure 5-10. Earthquake soil and water pressures**

The dynamic forces are (Figure 5-10a):

$$\Delta P_{E1} = k_h \left[ \frac{\gamma_m \, (h^2 - d_c^2)}{2 \, (\tan \alpha - \tan \beta)} \right] \tag{5-38}$$

$$\Delta P_{E2} = k_h \left[ \frac{(\gamma_s - \gamma_m) \, h_s^2}{2 \tan \alpha} \right] \tag{5-39}$$

$$\Delta P_E = \Delta P_{E1} + \Delta P_{E2} \tag{5-40}$$

The static lateral water force is:

$$P_w = \frac{1}{2} \gamma_w \, h_s^2 \tag{5-41}$$

Then the total driving force is:

$$P_E = P + P_w + \Delta P_E \tag{5-42}$$

where

$\gamma$ = the average effective unit weight of soil

$\gamma_m$ = moist unit weight of soil above water table

$\gamma_s$ = saturated unit weight of soil below water table

$\gamma_b$ = buoyant unit weight of soil below water table

$\gamma_w$ = unit weight of water

$d_c$ = depth of cohesive crack

$h$ = height of vertical face

$h_s$ = height of water table

$k_h$ = horizontal acceleration coefficient

*b. Resisting force.* The static and dynamic components for a resisting side wedge are illustrated in Figure 5-10b.

$p_s$ = the lateral static earth pressure at the water table

$p$ = the lateral static earth pressure at the bottom of the wedge

$2\,K_c\,c =$ the static lateral earth pressure at the top

$$p_s = K\,\gamma_m \left( \frac{\tan\alpha}{\tan\alpha - \tan\beta} \right) (h - h_s) + 2\,K_c\,c \tag{5-43}$$

$$p = K \left[ \gamma_m \left( \frac{\tan\alpha}{\tan\alpha - \tan\beta} \right) - (\gamma_m - \gamma_b)\,h_s \right] + 2\,K_c\,c \tag{5-44}$$

Then the total static lateral earth force is:

$$P = \frac{1}{2}\,(p_s + 2\,K_c\,c)\,(h - h_s) + \frac{1}{2}\,(p_s + p)\,h_s \tag{5-45}$$

where

$$K = \frac{1 + \tan\phi\,\cot\alpha}{1 - \tan\phi\,\tan\alpha} \tag{5-46}$$

$$K_c = \frac{1}{2\,\cos^2\alpha\,(\tan\alpha - \tan\beta)\,(1 - \tan\phi\,\tan\alpha)} \tag{5-47}$$

and

$$\alpha = \tan^{-1}\left( \frac{-\,C_1 + \sqrt{C_1^2 + 4\,C_2}}{2} \right) \tag{5-48}$$

$$C_1 = \frac{2\,\tan\phi\,(\tan\phi - k_h) + \dfrac{4\,c\,(\tan\phi - \tan\beta)}{\gamma\,h}}{A} \tag{5-49}$$

$$C_2 = \frac{\tan\phi\,(1 + \tan\phi\,\tan\beta) + (\tan\beta - k_h) + \dfrac{2\,c\,(1 + \tan\phi\,\tan\beta)}{\gamma\,h}}{A} \tag{5-50}$$

$$A = (1 + k_h\,\tan\phi)\,\tan\phi + \frac{2\,c\,(1 + \tan\phi\,\tan\beta)}{\gamma\,h} \tag{5-51}$$

The dynamic forces are (Figure 5-10b):

$$\Delta P_{E1} = k_h \left[ \frac{\gamma_m \ h^2}{2 \ (\tan \ \alpha \ - \ \tan \ \beta \ )} \right] \tag{5-52}$$

$$\Delta P_{E2} = k_h \left[ \frac{\gamma_s \ - \ \gamma_m \ ) \ h_s^2}{2 \ \tan \ \alpha} \right] \tag{5-53}$$

$$\Delta P_E = \Delta P_{E1} + \Delta P_{E2} \tag{5-54}$$

The static lateral water force is:

$$P_w = \frac{1}{2} \gamma_w \ h_s^2 \tag{5-55}$$

Then the total resisting force is:

$$P_E = P + P_w - \Delta P_E \tag{5-56}$$

*c. Inertia force of wall.* The inertial force of the structure, including that part of the soil above the heel or toe and any water that is not included in the earth wedges, is computed by multiplying the acceleration coefficient by the weight as determined above.

$$F = ma \left( \frac{g}{g} \right) = \frac{a}{g} \ W = k_h \ W \tag{5-57}$$

*d. Dynamic force due to water above ground level.* Water standing above ground can have its static pressure, acting against the structure, either increased or decreased due to seismic action. Figure 5-11 shows the pressures and forces from freestanding water due to seismic action. The dynamic force is given by Westergaard's (1933) equation as:

$$P_E = \left( \frac{2}{3} \right) \ C_E \ k_h \ h^2 \tag{5-58}$$

where

$C_E$ = a factor depending on the depth of water

$h$ = water depth in feet

$T$ = earthquake’s period of vibration in seconds.

Westergaard's approximate equation for $C_E$ is:

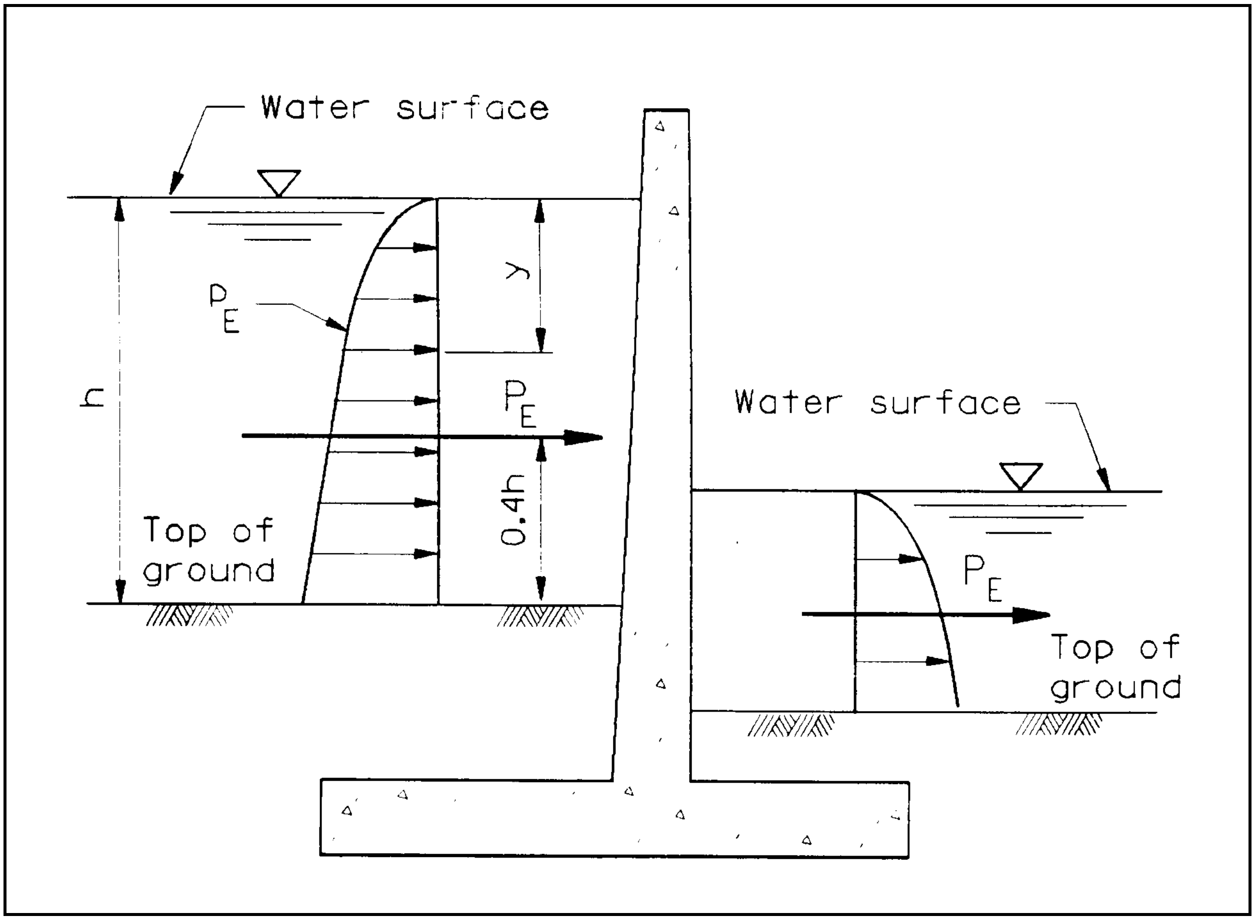



**Figure 5-11. Hydrodynamic forces for freestanding water**

$$C_E = \frac{0.051}{\sqrt{1 - 0.72\left(\frac{h}{1000\ T}\right)^2}} \tag{5-59}$$

Normally $C_E$ can be taken as 0.051. The pressure distribution is parabolic, and the pressure at any point $y$ below the top surface is:

$$p_E = C_E\ k_h \sqrt{h\ y} \tag{5-60}$$

The line of action for the force $P_E$ is 0.4 $h$ above the ground surface.

## 5-7. Surge and Wave Loads

These loads are critical in analyzing the stability of coastal protection structures but usually have little, if any, effect on the stability of inland structures.

*a. General criteria.* Wave and water level predictions for the analysis of structures should be determined with the criteria presented in the Shore Protection Manual 1984. Design forces acting on the structure should be determined for the water levels and waves predicted for the most severe fetch and the effects of shoaling, refraction, and diffraction. A distinction is made between the action of nonbreaking, breaking, and broken waves, where the methods recommended for calculation of wave forces are for vertical surfaces. Wave forces on other types of surfaces (sloping, stepped, curved, etc.) are not sufficiently understood to recommend general analytical design criteria. In any event, a coastal engineer should be involved in establishing wave forces for the design of critical structures.

*b. Wave heights.* Wave heights for design are obtained from the statistical distribution of all waves in a wave train and are defined as follows:

$H_S$ = average of the highest one-third of all waves

$H_1$ = 1.67 $H_S$ = average of highest 1 percent of all waves

$H_b$ = height of wave which breaks in water depth $d_b$

*c. Nonbreaking wave condition.* When the water depth is such that waves do not break, a nonbreaking condition exists. This occurs when the water depth at the structure is greater than approximately 1.5 times the maximum wave height. The $H_1$ wave shall be used for the nonbreaking condition. Design nonbreaking wave pressures shall be computed using the Miche-Rudgren Method, as described in Chapter 7 of the Shore Protection Manual (1984). Whenever the maximum stillwater level results in a nonbreaking condition, lower stillwater levels should be investigated for the possibilty that shallow water may produce breaking wave forces which are larger than the nonbreaking forces.

*d. Breaking wave condition.* The breaking condition occurs when the steepness of the wave and the bottom slope at the front of the structure have certain relationships to each other. It is commonly assumed that a structure positioned in a water depth, $d_s$, will be subject to the breaking wave condition if $D_s$ is equal to or less than 1.3 H, where H is the design wave height. Study of the breaking process indicates that this assumption is not always valid. The height of the breaking wave and its breaking point are difficult to determine, but breaker height can equal the water depth at the structure, depending on bottom slope and wave period. Detailed determination of breaker heights and distances for a sloping approach grade in front of the structure are given in the Shore Protection Manual 1984. Special consideration must be given to a situation where the fetch shoals abruptly near the structure, but at a distance more than approximately 0.7 of a wavelength away from the structure, and then maintains a constant water depth from that point to the wall. In this case, waves larger than the water depth can be expected to have broken at the abrupt shoaling point, allowing smaller, higher frequency waves to reach the structure. Design breaking wave pressure should be determined by the Minikin Method presented in Chapter 7 of the Shore Protection Manual (1984). Breaking-wave impact pressures occur at the instant the vertical force of the wave hits the structure and only when a plunging wave entraps a cushion of air against the structure. Because of this dependence on curve geometry, high impact pressures are infrequent against prototype structures; however, they must be recognized and considered in design. Also, since the impact pressures caused by breaking waves are of high frequency, their importance in design against sliding and rotational instability may be questionable relative to longer lasting, smaller dynamic forces.

*e. Broken wave condition.* Broken waves are those that break before reaching the structure, but near enough to have retained some of the forward momentum of breaking. The design breaker height in this case ($H_b$) is the highest wave that will be broken in the break zone. Design wave forces for this height should be determined by the method presented in Chapter 7 of the Shore Protection Manual (1984).

*f. Seepage pressures.* Seepage pressures shall be based on the elevation of the surge stillwater level.